ON POLYNOMIAL CONJECTURES OF NILPOTENT LIE GROUPS UNITARY REPRESENTATIONS

ALI BAKLOUTI

Abstract. Let G be a connected and simply connected nilpotent Lie group of Lie algebra $\mathfrak{g}$, K an analytic subgroup of G, χ a unitary character of G and π an irreducible unitary representation of G. In this setting, the orbit method allows to identify the unitary dual of G to the space of coadjoint orbits. Using the enveloping algebra of the complexified Lie algebra of $\mathfrak{g}$, we consider two algebras of differential operators $D\pi(G)^K$ and $D_\tau(G/K)$ associated respectively to the restriction $\pi|K$ of π to K and to the monomial representation $\tau = \mathrm{Ind}_G^K \chi$. Under the assumption that these representations are of finite multiplicities, the polynomial conjectures stating that $D\pi(G)^K$ and $D_\tau(G/K)$ are K-invariant polynomial rings hold. In this lecture, I will overview some history of the conjectures and some restrictive cases and talk about some steps of their proofs, for which the study of the geometry and the saturation of coadjoint orbits plays a crucial role.

Ali Baklouti. Department of Mathematics, Faculty of Sciences at Sfax, Route de Soukra, 3038, Sfax, Tunisia

Email address: Ali.Baklouti@usf.tn