Jordan $*$-derivations and n^{th}-power property in rings

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Let R be an associative ring with involution $'*'$. For any $x, y \in R$, as usual the symbols $x \circ y$ and $[x, y]$ will denote the anti-commutator $xy + yx$ and commutator $xy - yx$ and called Jordan product and Lie product, respectively. Recall that an additive map $D : R \to R$ is said to be a derivation if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$. Following [I. N. Herstein, Proc. Amer Math. Soc. 8 (1957), 1104-1110], an additive map $D : R \to R$ is called a *Jordan derivation* if $D(x^2) = D(x)x + xD(x)$ holds for all $x \in R$. Such a map, we call Jordan type. An additive map $D : R \to R$ is called a *Jordan $*$-derivation* if $D(x^2) = D(x)x^* + xD(x)$ holds for all $x \in R$, where R is a ring with involution $'*'$. For an element $a \in R$, it is easy to verify that the map $D : x \mapsto xa - ax^*$ for all $x \in R$, is a Jordan $*$-derivation. Such D is called an inner Jordan $*$-derivation (viz.; [S. Ali and N. A. Dar, Comm. Algebra 49(4) (2021), 1422-1430] and [T. K. Lee and Y. Zhou, J. Algebra Appl. 13(4)(2014)] for details and recent results).

The aim of this talk is to present some old and new results on Jordan $*$-derivations and its related maps in rings with involution obtained by applying theory of functional identties. Moreover, some examples and counter examples will be discussed for questions raised naturally. Finally, we conclude our talk with some open problems.

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