On the Irreducibility of Truncated Binomial Polynomials

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Abstract.
For positive integers $n \geq m$, let

$$P_{n,m}(x) := \sum_{j=0}^{m} \binom{n}{j} x^j = \binom{n}{0} + \binom{n}{1} x + \cdots + \binom{n}{m} x^m$$

be the truncated binomial expansion of $(1+x)^n$ consisting of all terms of degree $\leq m$. These polynomials arose in the investigation of Schubert calculus of Grassmannians. It is conjectured that for $n > m+1$, the polynomial $P_{n,m}(x)$ is irreducible. We confirm this conjecture when $2m \leq n < (m+1)^{10}$. Under explicit abc conjecture, for a fixed m, we give an explicit n_0 depending only on m such that $\forall n \geq n_0$, the polynomial $P_{n,m}(x)$ is irreducible. This is a joint work with P. Yadav.