On Generalization from Almost von Neumann Regular Rings to Almost von Neumann Regular Modules

Budi Surodjo[1, a)], Sri Wahyuni[1)], Fahreezan Sheraz Diyaldin[2)], Carolus Pasha L. J. H. [2)], Orlando Ferrari[2)]
[1)] Department of Mathematics, Universitas Gadjah Mada
[2)] Student of Mathematics Undergraduate Program, Universitas Gadjah Mada
[a)] Corresponding author: surodjo_b@ugm.ac.id

Abstract. A ring R is called almost von Neumann regular (AVN-ring for short) if for any two elements a and b in R, there exists a positive integer n such that the ideal $(a^n; b^n)$ is generated by an idempotent element. Recall that a ring R is called von Neumann regular if every finitely generated ideal is generated by an idempotent element. Clearly, a von Neumann regular ring is an AVN-ring. On the other hand, an R-module M is called von Neumann regular (VN-module for short) if for any m in M, $Rm=aM$ for some M-vn-regular element a in R (namely an element satisfy $aM=a^2M$). The definition von Neumann regular module is a generalization of the equivalent condition of R to be von Neumann regular ring if and only if $aR=a^2R$.

In this paper we will introduce the definition of almost von Neumann regular module (AVN-Module for short) as follows: an R-module M is called Almost von Neumann regular (AVN-module for short) if for any two elements a and b in R there exists a positive integer n such that $a^nM+b^nM=eM$ for some idempotent element of R. We will present some properties of AVN-module.

Keywords: von Neumann rings, von Neumann modules, and almost von Neumann regular rings.